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Applied Mathematics Letters 13 (2000) 55–62

Applied Mathematics Letters

www.elsevier.nl/locate/aml

A Class of Projection-Contraction Methods Applied to Monotone Variational Inequalities

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(Received May 1999; accepted June 1999)

Communicated by R. P. Agarwal

Abstract—First, an extension of the projection-contraction (PC) method is introduced, which generalizes a class of the existing PC methods, and then the extended projection-contraction (EPC) method is applied to the solvability of a class of general monotone variational inequalities.

Keywords—Extended projection-contraction method, Monotone quasivariational inequality, Projection method, Monotone mapping.

1. INTRODUCTION

In recent years, several iterative procedures [1–22] have been applied to the solvability of general variational inequalities. Among the most notable methods, especially the widely used projection method is more restrictive in the sense of operators involved (strongly monotone) than extragradient method [5] which requires the double projection formula and is easy to implement as it needs little storage and readily exploits any sparsity or separable structure in the operator set. As the extragradient method needs two function evaluation per iteration, the convergence requires only a solution to exist. Recently, Noor [8] extended the extragradient method to the case of a Hilbert space setting, while He [2,3], Solodov and Tseng [10], and Sun [11] applied a new class of projection-contraction (PC) methods to the solvability of a general class of monotone variational inequalities. The PC methods are simple and robust, and have the capacity to handle large problems with starting point. They may also be used to solve general convex quadratic programming problems. Moreover, numerical experiments indicate that these methods could very well be efficient for large sparse problems. Here, our plan is first to extend the PC method to the case of a Hilbert space setting, and then apply the extended PC (EPC) method to the solvability of a general class of monotone variational inequalities. On top of that, we achieve convergence of the approximate solutions under less restrictive assumptions.

Next, consider a real Hilbert space H with the inner product and norm, respectively, denoted by $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$. Let $T, g : H \to H$ be mappings on H, and $J : H \to P(H)$ a multivalued mapping, where $P(H)$ denotes the power set of H. We intend to consider the monotone quasivariational

0893-9659/00/$ - see front matter Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX
PII: S0893-9659(00)00096-3

inequality (MQVI) problem, determine an element $x \in H$ such that $g(x) \in J(x)$, and

$$\langle T(x), v - g(x)\rangle \geq 0, \qquad \text{for all } v \in J(x). \tag{1.1}$$

For $g \equiv I$ (the identity), the MQVI problem (1.1) reduces to, determine an element $x \in H$ such that $x \in J(x)$ and

$$\langle T(x), v - x\rangle \geq 0, \qquad \text{for all } v \in J(x). \tag{1.2}$$

Next, we recall some definitions and related properties concerning the work on hand.

PROPOSITION 1.1. *Let $T, g : H \to H$ be mappings on a Hilbert space H. Then, the following statements are equivalent.*

(i) *There exists a constant $r > 0$ such that*

$$\|T(x) - T(y)\|^2 \geq r^2\|g(x) - g(y)\|^2 + \|T(x) - T(y) - r(g(x) - g(y))\|^2, \qquad \textit{for all } x, y \in H.$$

(ii) *For each $x, y \in H$, we have*

$$\langle T(x) - T(y), g(x) - g(y)\rangle \geq r\|g(x) - g(y)\|^2,$$

where $r > 0$ is a constant, that is, T is $g - r$-strongly monotone.

Note that (ii) implies that T is $g - r$-expanding, that is,

$$\|T(x) - T(y)\| \geq r\|g(x) - g(y)\|.$$

For $r = 1$, T is said to be a g-expanding mapping, and for $r = 1$ and $g \equiv I$, T is called an expanding mapping.

For $g \equiv I$ in Proposition 1.1, we have the following.

PROPOSITION 1.2. *(See [13].) Let $T : H \to H$ be any mapping on H and $r > 0$ a constant. Then, the following statements are equivalent.*

(i) *For each $x, y \in H$, we have*

$$\|T(x) - T(y)\|^2 \geq r^2\|x - y\|^2 + \|T(x) - T(y) - r(x - y)\|^2.$$

(ii) $\langle T(x) - T(y), x - y\rangle \geq r\|x - y\|^2$, *for all $x, y \in H$.*

PROPOSITION 1.3. *Let $T : H \to H$ be a mapping on H. Then, the following statements are equivalent.*

(i) *There exists a constant $r > 0$ such that*

$$\|T(x) - T(y)\|^2 \geq r^2\|x - y\|^2 + \|T(x) - T(y) - r(x - y)\|^2, \qquad \textit{for all } x, y \in H.$$

(ii) *For each $x, y \in H$, we have*

$$\langle T(y), x - y\rangle \geq 0 \Rightarrow \langle T(x), x - y\rangle \geq r\|x - y\|^2,$$

where $r > 0$ is a constant, that is, T is strongly r-pseudomonotone.

PROPOSITION 1.4. *Let $T : H \to H$ be a mapping on H. Then, the following statements are equivalent.*

(i) *There exists a constant $r > 0$ such that*

$$\|T(x) - T(y)\|^2 + r^2\|g(x) - g(y)\|^2 \geq \|T(x) - T(y) - r(g(x) - g(y))\|^2, \quad \textit{for all } x, y \in H.$$

(ii) $\langle T(x) - T(y), r(g(x) - g(y))\rangle \geq 0$, *for all $x, y \in H$, where $r > 0$ is a constant, that is, T is $g - r$-monotone, where $g : H \to H$ is any mapping on H.*

For $r = 1$ in Proposition 1.4, we have the following.

PROPOSITION 1.5. *Let $T : H \to H$ be a Hilbert space mapping. Then, the following statements are equivalent.*

(i) *For $x, y \in H$, we have*

$$\|T(x) - T(y)\|^2 + \|g(x) - g(y)\|^2 \geq \|T(x) - T(y) - (g(x) - g(y))\|^2.$$

(ii) *T is g-monotone, that is,*

$$\langle T(x) - T(y), g(x) - g(y)\rangle \geq 0, \qquad \text{for all } x, y \in H.$$

Recently, the author of [20] introduced an alternative notion to the existing concept of the cocoercivity of a Hilbert space mapping, which appears to be application-oriented in the sense of algorithmic applications, in the following manner.

A Hilbert space mapping $T : H \to H$ is said to be α-cocoercive if there exists a constant $\alpha > 0$ such that

$$\|x - y\|^2 \geq \alpha^2\|T(x) - T(y)\|^2 + \|\alpha(T(x) - T(y)) - (x - y)\|^2, \qquad \text{for all } x, y \in H.$$

An operator $T : H \to H$ is called α-cocoercive [6] if for all $x, y \in H$, we have

$$\langle T(x) - T(y), x - y\rangle \geq \alpha\|T(x) - T(y)\|^2,$$

where $\alpha > 0$ is a constant.

Note that every strongly monotone and Lipschitz continuous mapping is cocoercive, and every cocoercive mapping is monotone.

PROPOSITION 1.6. *Let $T : H \to H$ be any mapping on H. Then, we have the following equivalent statements.*

(i) *For each $x, y \in H$ and for a constant $\alpha > 0$, we have*

$$\|x - y\|^2 \geq \alpha^2\|T(x) - T(y)\|^2 + \|\alpha(T(x) - T(y)) - (x - y)\|^2.$$

(ii) *There exists a constant $\alpha > 0$ such that*

$$\langle T(x) - T(y), x - y\rangle \geq \alpha\|T(x) - T(y)\|^2, \qquad \text{for all } x, y \in H.$$

PROPOSITION 1.7. *Let $T, g : H \to H$ be any mappings on H. Then, the following statements are equivalent.*

(i) *For each $x, y \in H$ and for a constant $\alpha > 0$, we have*

$$\|g(x) - g(y)\|^2 \geq \alpha^2\|T(x) - T(y)\|^2 + \|\alpha(T(x) - T(y)) - (g(x) - g(y))\|^2.$$

(ii) *There exists a constant $\alpha > 0$ such that*

$$\langle T(x) - T(y), g(x) - g(y)\rangle \geq \alpha\|T(x) - T(y)\|^2, \qquad \text{for all } x, y \in H.$$

An operator $T : H \to H$ is said to be $g-k$-Lipschitz continuous if

$$\|T(x) - T(y)\| \le k\|g(x) - g(y)\|, \qquad \text{for all } x, y \in H,$$

where $k \ge 0$ is a constant and $g : H \to H$ is any mapping on H. This implies that

$$\langle T(x) - T(y), g(x) - g(y)\rangle \le k\|g(x) - g(y)\|^2.$$

For $g \equiv I, T$ is called a k-Lipschitz continuous (or Lipschitz continuous) mapping.

An operator $T : H \to H$ is said to be $g-k$-pseudocontraction [23] if there exists a constant $k > 0$ such that

$$\langle T(x) - T(y), g(x) - g(y)\rangle \le k\|g(x) - g(y)\|^2, \qquad \text{for all } x, y \in H,$$

where $g : H \to H$ is any mapping on H.

For $g = I$, the identity mapping, $T : H \to H$ is called an k-pseudocontraction.

LEMMA 1.1. *(See [4].) For an element $z \in H$, an element $x \in K$, and*

$$\langle x - z, y - x\rangle \ge 0, \qquad \text{for all } y \in K \text{ if and only if } x = P_K(z),$$

where K is a closed convex subset of H and P_K is the projection of H onto K.

Korpelevich [5] introduced the following extragradient method:

$$x^{k+1} = P_K\left[x^k - \beta F\left(P_K\left[x^k - \beta F\left(x^k\right)\right]\right)\right],$$

where $F : K \to K$ is monotone and L-Lipschitz continuous, P_K is the projection of H onto a closed convex subset K of H, and $\beta \in (0, 1/L)$ is a constant.

2. EXTENDED PROJECTION-CONTRACTION METHOD

This section is intended to deal with the approximation-solvability of the MQVI problem (1.1) along with some special cases of interest.

PROPOSITION 2.1. *Let H be a real Hilbert space and $T, g : H \to H$ be any mappings on H. Suppose that $J : H \to P(H)$ is a multivalued mapping such that $J(x)$ is closed convex for all $x \in H$. Then, the following statements are equivalent.*

(i) *An element $x \in H$ is a solution of the MQVI problem (1.1).*
(ii) *$g(x) = P_{J(x)}[g(x) - tT(x)]$ for a constant $t > 0$,*

where $P_{J(x)}$ is the projection of H onto $J(x)$.

PROPOSITION 2.2. *Let H be a real Hilbert space, $T, g : H \to H$ be any mappings, and $J : H \to P(H)$ a multivalued mapping such that $J(x)$ is closed convex for all $x \in H$. Then, the following statements are equivalent.*

(i) *An element $x \in H$ is a solution of the MQVI problem (1.1).*
(ii) *An element $x \in H$ is a fixed point of $Ft : H \to H$ defined by*

$$Ft(v) = v - g(v) + P_{J(v)}[g(v) - tT(v)], \qquad \text{for } v \in H,$$

where $t > 0$ is a constant.

PROPOSITION 2.3. *Let H be a real Hilbert space, and $T, g : H \to H$ any mappings. Suppose that $J : H \to P(H)$ is a multivalued mapping such that $J(u) = m(u) + K$ for $u \in H$, where $m : H \to H$ is a single-valued mapping and K is a closed convex subset of H. Then, the following statements are equivalent.*

(i) *An element $x \in H$ is a solution of the MQVI problem (1.1).*
(ii) *$g(u) = m(u) + P_K[g(u) - m(u) - tT(u)]$, for all $u \in H$,*

where $t > 0$ is a constant.

In light of Propositions 2.1 and 2.2, if an element $u \in H$ is a solution of the MQVI problem (1.1), then we have

$$g(u) = P_{J(u)}[g(u) - tT(u)] \qquad \text{and} \qquad u = u - g(u) + P_{J(u)}[g(u) - tT(u)].$$

For a constant $t > 0$, we define the residue function $R(u, t)$ by

$$R(u,t) = g(u) - P_{J(u)}[g(u) - tT(u)],$$

and the search direction function $d(u, t)$ by

$$d(u,t) = R(u,t) - t\left\{T(u) - T\left(u - g(u) + P_{J(u)}[g(u) - tT(u)]\right)\right\}.$$

Clearly, an element $u \in H$ with $g(u) \in J(u)$ is a solution of the MQVI problem (1.1) if and only if $u \in H$ such that $g(u) \in J(u)$ and $R(u, t) = 0$.

THEOREM 2.1. *Let H be a real Hilbert space, $T, g : H \to H$ any mappings, and $J : H \to P(H)$ a multivalued mapping such that $J(x)$ is closed convex for all $x \in H$. Let T be a k-pseudocontraction. Then,*

$$\langle R(u,t), d(u,t)\rangle \geq (1 - kt)\|R(u,t)\|^2.$$

THEOREM 2.2. *Let H be a real Hilbert space, $T, g : H \to H$ any mappings on H, and $J : H \to P(H)$ a multivalued mapping such that $J(x)$ is a closed convex subset of H for all $x \in H$. Let T be a k-pseudocontraction and g-monotone. Suppose that $u' \in H$ is a solution of the MQVI problem (1.1). Then,*

$$\langle g(u) - g(u'), d(u,t)\rangle \geq (1 - kt)\|R(u,t)\|^2.$$

PROOF. Assume $u' \in H$ is a solution of the MQVI problem (1.1). Then, we have

$$\langle T(u'), v - g(u')\rangle \geq 0, \qquad \text{for all } v \in J(u'). \tag{2.1}$$

If we replace v in (2.1) by $g(u)$, then for a constant $t > 0$, we have

$$t\langle T(u'), g(u) - g(u')\rangle \geq 0, \qquad \text{for all } g(u) \in J(u'). \tag{2.2}$$

Taking $z = g(u) - tT(u)$, $x = P_{J(u)}[g(u) - tT(u)]$, and $y = g(u')$ in Lemma 1.1, we get

$$\left\langle P_{J(u)}[g(u) - tT(u)] - g(u) + tT(u), g(u') - P_{J(u)}[g(u) - tT(u)]\right\rangle \geq 0$$

or

$$\left\langle R(u,t) - tT(u), P_{J(u)}[g(u) - tT(u)] - g(u')\right\rangle \geq 0. \tag{2.3}$$

It follows from (2.2) and (2.3) that

$$\left\langle R(u,t) - t(T(u) - T(u')), P_{J(u)}[g(u) - tT(u)] - g(u')\right\rangle \geq 0. \tag{2.4}$$

Since T is g-monotone, we have

$$t\langle T(u) - T(u'), g(u) - g(u')\rangle \geq 0, \tag{2.5}$$

where $g(u) = P_{J(u)}[g(u) - tT(u)]$.

Combining (2.4) and (2.5), we arrive at

$$\left\langle R(u,t), P_{J(u)}[g(u) - tT(u)] - g(u')\right\rangle \geq 0$$

or

$$\langle R(u,t), g(u) - R(u,t) - g(u') \rangle \geq 0$$

or

$$\left\langle R(u,t) - t\left\{T(u) - T\left(u - g(u) + P_{J(u)}[g(u) - tT(u)]\right)\right\}, g(u) - g(u') - R(u,t)\right\rangle \geq 0.$$

This implies that

$$\begin{aligned}&\left\langle g(u) - g(u'), R(u,t) - t\left\{T(u) - T\left(u - g(u) + P_{J(u)}[g(u) - tT(u)]\right)\right\}\right\rangle \\ \geq &\left\langle R(u,t), R(u,t) - t\left\{T(u) - T\left(u - g(u) + P_{J(u)}[g(u) - tT(u)]\right)\right\}\right\rangle \geq (1-kt)\|R(u,t)\|^2. \quad \blacksquare\end{aligned}$$

Now, we consider the extended projection-contraction (EPC) method for the solvability of the MQVI problem (1.1).

ALGORITHM 2.1. For $u_0 \in H$ and $g(u_0) \in J(u_0)$, compute an approximate solution u_{n+1} by an iterative algorithm (for a positive stepsize $\gamma \in (0,2)$)

$$g(u_{n+1}) = g(u_n) - \gamma\left(g(u_n) - P_{J(u_n)}[g(u_n) - tT(u_n)]\right), \qquad \text{for } n \geq 0,$$

where u_n is not a member of the solution set for the MQVI problem (1.1).

For $g = I$ (the identity) in Algorithm 2.1, we have the following.

ALGORITHM 2.2. For a given element $u_0 \in J(u_0)$, and for u_n not belonging to the solution set of the MQVI problem (1.1), we have

$$u_{n+1} = u_n - \gamma\left(u_n - P_{J(u_n)}[u_n - tT(u_n)]\right), \qquad \text{for } n \geq 0,$$

where $\gamma \in (0,2)$ is a positive stepsize.

For $J(u) = m(u) + K$, where $m : H \to H$ is any mapping and K is a closed convex subset of H, we have the algorithm.

ALGORITHM 2.3. For an arbitrarily chosen element $u_0 \in H$ and $g(u_0) \in K$, the sequence $\{u_n\}$ is generated by an iterative scheme

$$g(u_{n+1}) = g(u_n) - \gamma\left(g(u_n) - m(u_n) - P_K[g(u_n) - tT(u_n) - m(u_n)]\right), \qquad \text{for } n \geq 0,$$

where $\gamma \in (0,2)$ is a positive stepsize.

For $J(u) \equiv K$, a closed convex subset of H, we have the following.

ALGORITHM 2.4. For an arbitrarily chosen element $u_0 \in H$ such that $g(u_0) \in K$, a sequence $\{u_n\}$ is generated by an iterative procedure

$$g(u_{n+1}) = g(u_n) - \gamma(g(u_n) - P_K[g(u_n) - tT(u_n)]), \qquad \text{for } n \geq 0.$$

THEOREM 2.3. *Let H be a real finite Hilbert space, and $T, g : H \to H$ be any mappings. Suppose that $J : H \to P(H)$ is a multivalued mapping such that $J(x)$ is closed convex for all $x \in H$ and the following assumptions hold:*

(i) *T is g-monotone and k-Lipschitz continuous;*
(ii) *g is an expanding mapping, that is, $\|g(u) - g(v)\| \geq \|u - v\|$, for all $u, v \in H$;*
(iii) *g is p-Lipschitz continuous.*

Then, for an arbitrarily chosen element $x_0 \in H$, for a sequence $\{x_n\}$ generated by an iterative Algorithm 2.1, and for a solution $u' \in H$ of the MQVI problem (1.1), we have

$$\|g(u_{n+1}) - g(u')\|^2 \leq \|g(u_n) - g(u')\|^2 - \gamma(2 - \gamma - 2kt)\|R(u_n, t)\|^2. \tag{2.6}$$

PROOF. For an element $u' \in H$, a solution of the MQVI problem (1.1), we have

$$\begin{aligned}
\|g(u_{n+1}) - g(u')\|^2 &= \left\|g(u_n) - g(u') - \gamma\left(g(u_n) - P_{J(u)}[g(u_n) - tT(u_n)]\right)\right\|^2 \\
&= \|g(u_n) - g(u')\|^2 - 2\gamma\left\langle g(u_n) - g(u'), g(u_n) - P_{J(u_n)}[g(u_n) - tT(u_n)]\right\rangle \\
&\quad + \gamma^2\left\|g(u_n) - P_{J(u_n)}[g(u_n) - tT(u_n)]\right\|^2 \\
&= \|g(u_n) - g(u')\|^2 - 2\gamma\left\langle g(u_n) - g(u'), R(u_n, t)\right. \\
&\quad \left. -t\left\{T(u_n) - T\left(u_n - g(u_n) + P_{J(u_n)}[g(u_n) - tT(u_n)]\right)\right\}\right\rangle \\
&\quad + \gamma^2\left\|g(u_n) - P_{J(u_n)}[g(u_n) - tT(u_n)]\right\|^2 \\
&\leq \|g(u_n) - g(u')\|^2 - 2\gamma(1 - kt)\|R(u_n, t)\|^2 \\
&\quad + \gamma^2\left\|g(u_n) - P_{J(u_n)}[g(u_n) - tT(u_n)]\right\|^2 \\
&= \|g(u_n) - g(u')\|^2 - \gamma(2 - \gamma - 2kt)\|R(u_n, t)\|^2. \qquad \blacksquare
\end{aligned}$$

Next, we discuss the convergence of $\{u_n\}$ to a solution u' of the MQVI problem (1.1). It follows from Theorem 2.3 that $\{\|g(u_n) - g(u')\|\}$ is a decreasing sequence, and as a result, there exists a number $m \geq 0$ such that

$$\lim_{n\to\infty} \|g(u_n) - g(u')\| = m \geq 0,$$

and hence, we have

$$\lim_{n\to\infty} \|R(u_n, t)\| = 0.$$

Since g is an expanding mapping, it implies that (as $n \to \infty$)

$$\|u_n - u'\|^2 \leq \|g(u_n) - g(u')\|^2 \to m^2.$$

Thus, $\{u_n\}$ generated by the EPC method is a bounded sequence. Let u^* be a cluster point of $\{u_n\}$. Then, there exists a subsequence $u_{n_j} \to u^*$. Since g, T are continuous, it implies that

$$R(u^*) = \lim_{n\to\infty} R\left(u_{n_j}, t\right) = 0.$$

Thus, u^* is a solution of the MQVI problem (1.1). Since (2.6) holds for any solution u' of the MQVI Problem (1.1), we can replace u' by u^* in (2.6) and can have that the sequence $g(u_n) \to g(u^*)$. Since g is an expanding mapping, we have

$$\lim_{n\to\infty} \|u_n - u^*\| \leq \lim_{n\to\infty} \|g(u_n) - g(u^*)\| = 0,$$

that is, $u_n \to u^*$. $\blacksquare$

THEOREM 2.4. *Let $u' \in H$ be a solution of the MQVI problem (1.1). Let $T, g : H \to H$ be any mappings such that*

(i) *T is g-monotone and k-Lipschitz continuous;*
(ii) *g is expanding and p-Lipschitz continuous.*

Suppose that $\{u_n\}$ is a sequence generated by Algorithm 2.3. Then, $\{u_n\}$ converges to a solution of the MQVI problem (1.1).

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